

# Higgs Boson Mass, Sparticle Spectrum and Little Hierarchy Problem in Extended MSSM

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## Abstract

We investigate the impact of TeV-scale matter belonging to complete vectorlike multiplets of unified groups on the lightest Higgs boson in the MSSM. We find that consistent with perturbative unification and electroweak precision data the mass  $m_h$  can be as large as 160 GeV. These extended MSSM models can also render the little hierarchy problem less severe, but only for lower values of  $m_h (\lesssim 125)$  GeV. We present estimates for the sparticle mass spectrum in these models.

## 1 Introduction

The LEP2 lower bound  $m_h \geq 114.4$  GeV [1] on the Standard Model (SM) Higgs boson mass poses a significant challenge for the minimal supersymmetric standard model (MSSM). With the tree level upper bound of  $M_Z$  on the mass of the (lightest) SM-like Higgs boson in MSSM, significant radiative corrections are required to lift this mass above the LEP2 bound. This situation has been further exacerbated by the most recently quoted value of  $172.6 \pm 1.4$  GeV for the top quark pole mass [2], significantly lower from earlier values which not so long ago were closer to 176 GeV and higher [3]. With radiative corrections proportional to the fourth power of  $m_t$ , this leads to a reduced value for  $m_h$  unless the magnitude of some MSSM parameters such as the stop mass  $m_{\tilde{t}}$  (or  $M_S$ ) and the soft trilinear parameter  $A_t$  are suitably increased. Values of  $m_h$  of around 123 GeV or so require stop masses as well as  $|A_t|$  close to the TeV scale or higher. Such large values, in turn, lead to the so-called *little hierarchy* problem [4] because, when dealing with radiative electroweak symmetry breaking, TeV scale quantities must conspire to yield the electroweak mass scale  $M_Z$ .

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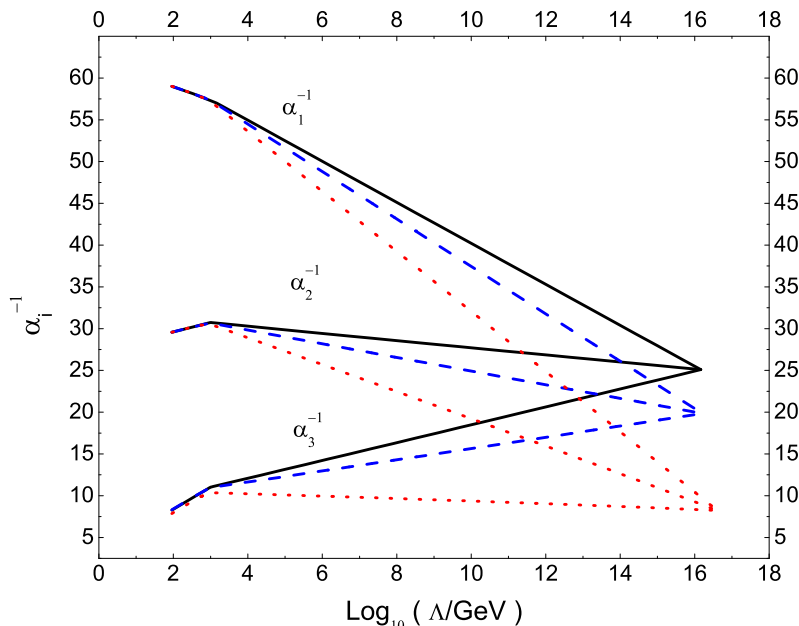


Figure 1: Gauge coupling evolution with the effective SUSY breaking scale  $M_S = 1$  TeV and  $\tan\beta = 10$ . Solid lines correspond to MSSM. Dashed lines correspond to MSSM+ $5 + \bar{5}$ . Dotted lines are for MSSM + $10 + \bar{10}$ . The vectorlike masses for all these cases are set equal to 500 GeV.

In this paper we address these two related conundrums of the MSSM by introducing TeV scale vectorlike matter superfields which reside in complete  $SU(5)$  or  $SO(10)$  multiplets. Such complete multiplets, it is well known, do not spoil unification of the MSSM gauge couplings. We illustrate this in Figure 1, where the gauge coupling evolution is compared, using two loop RGEs, for the case of MSSM plus complete multiplets  $10 + \bar{10}$  and  $5 + \bar{5}$  of  $SU(5)$ . If these vectorlike matter fields do not acquire Planck-scale masses, it appears quite plausible that they will end up order TeV masses.  $R$ -symmetries, for instance, can forbid Planck scale masses, but allow TeV scale masses proportional to the SUSY breaking scale. The Higgs(ino) mass term (the  $\mu$  parameter) for the  $H_u - H_d$  superfields is an example where this happens already in the MSSM [5]. For the vectorlike matter to have any significant effect on the ‘upper’ bounds on  $m_h$ , it is crucial that they have masses of order TeV, otherwise their effects on  $m_h$  will decouple.

In studying this possibility of extending the MSSM, we employ the perturbativity and GUT unification constraints. It turns out that perturbative unification can be maintained if we introduce either (i) one pair of  $(10 + \bar{10})$ , or (ii) up to four pairs of  $(5 + \bar{5})$ , or (iii) one set of  $(10 + \bar{10} + 5 + \bar{5})$  [7], where the representations refer to  $SU(5)$ . In addition, any number of SM singlet fields are also allowed. Some particles in these new supermultiplets couple to the MSSM Higgs doublet  $H_u$ , and with masses of order  $0.5 - 1$  TeV, their radiative contributions alone can lift  $m_h$  to values as high as 160 GeV. This is achieved without requiring the standard MSSM particles to be much heavier than their present experimental lower bounds. We explore the impact of the additional multiplets on the MSSM parameter space, and obtain the low energy particle spectrum in the mSUGRA framework. A comparison is presented, using semi-analytic

estimates, between the minimal and extended MSSM sparticle spectrum. The impact of these new particles on the little hierarchy problem is also discussed.

With the inclusion of these new vectorlike particles, the lightest Higgs boson mass, as previously noted, can be significantly increased. However, we find that resolving the little hierarchy problem is somewhat more tricky. The new Yukawa couplings of  $H_u$  to vectorlike matter, which helps in raising  $m_h$ , also has the effect of raising the soft Higgs mass parameter  $m_{H_u}^2$ , which could exacerbate the little hierarchy problem. If the new Yukawa couplings of  $H_u$  are relatively small, the little hierarchy problem improves relative to the MSSM, since the cumulative effect of the top Yukawa coupling  $y_t$  on  $m_{H_u}^2$  becomes smaller than in MSSM. This comes about since  $y_t$  has a smaller value at the GUT scale ( $y_t \sim 0.15$ ) compared to the MSSM case ( $y_t \sim 0.5$ ). Thus we identify two regions of the parameter space as being of special interest: one where the little hierarchy problem becomes worse than in MSSM, but where  $m_h \sim 130 - 160$  GeV can be achieved, and another where the little hierarchy problem is relaxed, but where  $m_h \lesssim 125$  GeV. The latter possibility appears to us to be quite interesting, as it assumes MSSM sparticle masses to be moderate, of order  $200 - 500$  GeV.

## 2 New vectorlike matter and precision constraints

It is well known that one can extend the matter sector of the MSSM and still preserve the beautiful result of gauge coupling unification provided that the additional matter superfields fall into complete multiplets of any unified group which contains the SM, such as  $SU(5)$ . Such extended scenarios with TeV scale matter multiplets are well motivated. Within string theory, for instance, one often finds light (TeV scale) multiplets in the spectrum [6], and even within the framework of GUTs one can find extra complete multiplets with masses around the TeV scale [7].

An important constraint on GUT representations and how many there can be at low ( $\sim$  TeV) scale comes from the perturbativity condition, which requires that the three MSSM gauge couplings remain perturbative up to  $M_G$ . One finds that there are several choices to satisfy this constraint: (i) one pair of  $(10 + \overline{10})$ , (ii) up to 4 pairs of  $(5 + \overline{5})$ 's, or (iii) the combination,  $(5 + \overline{5} + 10 + \overline{10})$ . Here all representations refer to multiplets of  $SU(5)$ . In addition, any number of MSSM gauge singlets can be added without sacrificing unification or perturbativity. Option (iii), along with a pair of MSSM gauge singlets fits nicely in  $SO(10)$  models.

Cases (i) and (iii) have been studied before in the literature. For example, the authors in [8] conclude that the mass of the lightest CP even Higgs mass could be pushed up to 180 GeV, consistent with all perturbativity constraints. When updated to account for the recent electroweak precision data, specifically the  $T$  parameter, and the current value of the top quark mass, and improved to include two-loop RGE effects and finite corrections to the Higgs boson mass, we find that these scenarios admit  $m_h$  only as large as 160 GeV, which is significantly smaller than the bounds in [8].

It is clear that new matter will contribute at one loop level to CP-even Higgs mass if there is direct coupling among new matter and the MSSM Higgs field. In case (i), a new couplings  $10 \cdot 10 \cdot H_u$  is allowed, analogous to the top-quark Yukawa coupling, but involving the charge  $2/3$  quark from the 10-plet. (Here we use for simplicity  $SU(5)$  notation, but with

the understanding that  $H_u$  and  $H_d$  are not complete multiplets of  $SU(5)$ .) This new Yukawa coupling can modify the upper limit on  $m_h$ , which we will study in detail, taking into account perturbativity constraints. By itself, case (ii) does not allow for any new Yukawa coupling unless the new states in the  $\bar{5}$  are mixed with the usual  $d^c$ -quarks and lepton doublets. Such a possibility is even more strongly constrained (by flavor violation and unitarity of the CKM matrix, among others), and so we will forbid all such mixing. Once we add gauge singlets 1, couplings such as  $\bar{5}H_u1$  are allowed (Only the lepton-like doublets from  $\bar{5}$  will be involved in this Yukawa coupling.) We will analyze the effects of such couplings on  $m_h$  in detail. Case (iii) is a combination of (i) and (ii), which will also be studied in detail.

There are constraints on the couplings and masses of new matter fields. Most important are the constraints from the  $S$  and  $T$  parameters which limit the number of additional *chiral* generations. Consistent with these constraints, one should add new matter which is predominantly vectorlike.

In the limit where the vectorlike mass is much heavier than the chiral mass term (mass term arising from Yukawa coupling to the Higgs doublets), the contribution to the  $T$  parameter from a single chiral fermion is approximately [9]:

$$\delta T = \frac{N(\kappa v)^2}{10\pi \sin^2 \theta_W m_W^2} \left[ \left( \frac{\kappa v}{M_V} \right)^2 + O \left( \frac{\kappa v}{M_V} \right)^4 \right], \quad (1)$$

where  $\kappa$  is the new chiral Yukawa coupling,  $v$  is VEV of the corresponding Higgs field, and  $N$  counts the additional number of  $SU(2)$  doublets. For instance,  $N = 3$  when  $10 + \bar{10}$  is considered at low scale, while  $N = 1$  for the  $5 + \bar{5}$  case. From precision electroweak data  $T \leq 0.06(0.14)$  at 95% CL for  $m_h = 117$  GeV (300 GeV) [16]. We will take  $\delta T < 0.1$  as a realistic bound and apply it in our analysis. We then see from Eq. (1) that with  $M_V$  around 1 TeV, the Yukawa coupling  $\kappa$  can be  $O(1)$ .

## 3 Higgs mass bound

### 3.1 MSSM + $10 + \bar{10}$

The representation  $10 + \bar{10}$  of  $SU(5)$  decomposes under the MSSM gauge symmetry as follows:

$$\begin{aligned} 10 + \bar{10} = & Q_{10} \left( 3, 2, \frac{1}{6} \right) + \bar{Q}_{10} \left( \bar{3}, 2, -\frac{1}{6} \right) + U_{10} \left( \bar{3}, 1, -\frac{2}{3} \right) + \bar{U}_{10} \left( 3, 1, \frac{2}{3} \right) \\ & + E_{10} (1, 1, 1) + \bar{E}_{10} (1, 1, -1). \end{aligned} \quad (2)$$

We assume for the vectorlike matter  $10 + \bar{10}$  the same  $R$  parity as the MSSM Higgs chiral superfields. So there is no mixing of this new matter with quarks, but they couple to the Higgs doublets. The contribution to the superpotential from these couplings is

$$W = \kappa_{10} Q_{10} U_{10} H_u + \kappa'_{10} \bar{Q}_{10} \bar{U}_{10} H_d + M_V (\bar{Q}_{10} Q_{10} + \bar{U}_{10} U_{10} + \bar{E}_{10} E_{10}), \quad (3)$$

where, for simplicity, we have taken a common vectorlike mass (at the GUT scale  $M_G$ ). Thus the up quark-like pieces of the 10 and  $\bar{10}$  get Dirac *and* vectorlike masses, while leaving the

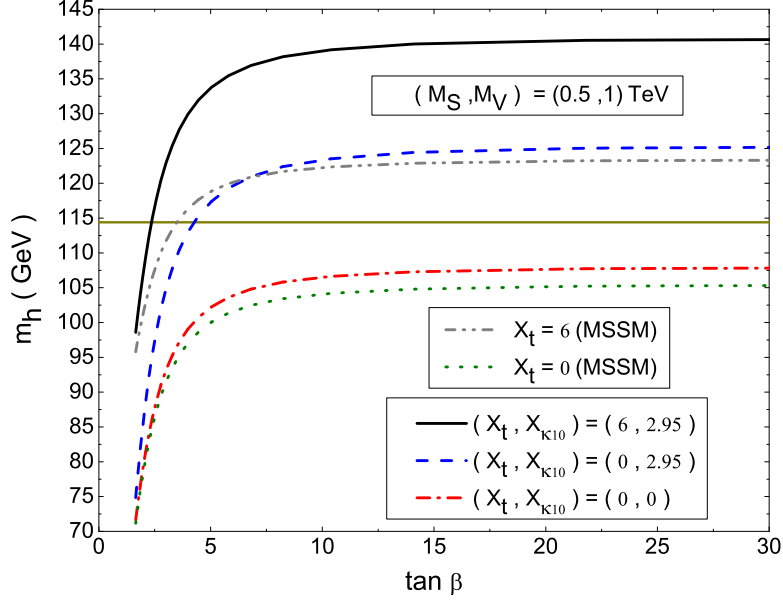


Figure 2: Upper bounds for the lightest CP-even Higgs boson mass vs  $\tan\beta$ , for different maximal and minimal values of  $X_t$ ,  $X_{\kappa_{10}}$ , with  $M_S = 500$  GeV,  $M_V = 1$  TeV and  $M_t = 172.6$  GeV. Dotted line corresponds to MSSM ( $X_t = 0$ ). Dashed-double dotted line describes MSSM with ( $X_t = 6$ ). Dashed-dotted curve is for MSSM +10 +  $\overline{10}$ .  $\kappa_{10} \approx 2$  at  $M_G$ . Dashed line shows Higgs mass with  $X_{10} = 2.95$  and  $X_t = 0$ . Solid line corresponds to  $X_{10} = 2.95$  and  $X_t = 6$ . The solid horizontal line denotes the LEP2 bound  $m_h = 114.4$  GeV.

$E_{10}$ -lepton-like pieces with only vectorlike masses. We assume that  $\kappa_{10} \gg \kappa'_{10}$  because the contribution coming from the coupling  $\kappa'_{10}$  reduces the light higgs mass similar to what we have with bottom Yukawa contribution which becomes prominent for large  $\tan\beta$  [10].

Employing the effective potential approach we calculate the additional contribution from the vectorlike particles to the CP even Higgs mass at one loop level. A similar calculation was carried out in ref. [11].

$$[m_h^2]_{10} = -M_Z^2 \cos^2 2\beta \left( \frac{3}{8\pi^2} \kappa_{10}^2 t_V \right) + \frac{3}{4\pi^2} \kappa_{10}^4 v^2 \sin^2 \beta \left[ t_V + \frac{1}{2} X_{\kappa_{10}} \right], \quad (4)$$

where we have assumed  $M_V \gg M_D$ . The corrected expression for  $X_{\kappa_{10}}$  (compare result in ref. [8]) is given as follows

$$X_{\kappa_{10}} = \frac{4\tilde{A}_{\kappa_{10}}^2 (3M_S^2 + 2M_V^2) - \tilde{A}_{\kappa_{10}}^4 - 8M_S^2 M_V^2 - 10M_S^4}{6(M_S^2 + M_V^2)^2} \quad (5)$$

and

$$t_V = \log \left( \frac{M_S^2 + M_V^2}{M_V^2} \right), \quad (6)$$

where  $\tilde{A}_{\kappa_{10}} = A_{\kappa_{10}} - \mu \cot \beta$ ,  $A_{\kappa_{10}}$  is the  $Q_{10} - U_{10}$  soft mixing parameter and  $\mu$  is the MSSM Higgs bilinear mixing term.  $M_S \simeq \sqrt{m_{\tilde{Q}_3} m_{\tilde{U}_3^c}}$ , where  $m_{\tilde{Q}_3}$  and  $m_{\tilde{U}_3^c}$  are the stop left and stop right soft SUSY breaking masses at low scale.

For completeness we present the leading 1- and 2- loop contributions to the CP-even Higgs boson mass in the MSSM [12, 13]

$$\begin{aligned} [m_h^2]_{MSSM} &= M_Z^2 \cos^2 2\beta \left( 1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} t \right) \\ &+ \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[ t + \frac{1}{2} X_t + \frac{1}{(4\pi)^2} \left( \frac{3}{2} \frac{m_t^2}{v^2} - 32\pi\alpha_s \right) (X_t t + t^2) \right], \end{aligned} \quad (7)$$

where

$$t = \log \left( \frac{M_S^2}{M_t^2} \right), \quad X_t = \frac{2\tilde{A}_t^2}{M_S^2} \left( 1 - \frac{\tilde{A}_t^2}{12M_S^2} \right). \quad (8)$$

Also  $\tilde{A}_t = A_t - \mu \cot \beta$ , where  $A_t$  denotes the stop left and stop right soft mixing parameter.

In our model for the light Higgs mass we have

$$m_h^2 = [m_h^2]_{MSSM} + [m_h^2]_{10}. \quad (9)$$

From Eq. (4) we can see that the Higgs mass is very sensitive to the value of  $\kappa_{10}$ , which we cannot take to be arbitrary large because the theory should be perturbative up to GUT scale. We therefore should solve the following RGE for  $\kappa_{10}$  to make sure that it remains perturbative up to the GUT scale:

$$\begin{aligned} \frac{d\kappa_{10}}{dt} &= \frac{\kappa_{10}}{2(4\pi)^2} \left( \left( \frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 - 6\kappa_{10}^2 - 3y_t^2 \right) \right. \\ &- \frac{1}{(4\pi)^2} \left( \frac{3913}{450} g_1^4 + \frac{33}{2} g_2^4 + \frac{128}{9} g_3^4 + g_1^2 g_2^2 + \frac{136}{45} g_1^2 g_3^2 + 8g_2^2 g_3^2 + \left( \frac{2}{5} g_1^2 + 6g_2^2 \right) \kappa_{10}^2 \right. \\ &+ \left. \left. \left( \frac{4}{5} g_1^2 + 16g_3^2 \right) (y_t^2 + \kappa_{10}^2) - 9(y_t^4 + \kappa_{10}^4) - 9\kappa_{10}^2 (y_t^2 + \kappa_{10}^2) - 4\kappa_{10}^4 \right) \right), \end{aligned} \quad (10)$$

where  $g_3$ ,  $g_2$  and  $g_1$  are strong, weak and hypercharge gauge couplings respectively and  $y_t$  denotes the top Yukawa coupling. Because the new matter couples to  $H_u$  (see Eq. (3)) there are additional contributions to the RGE for  $y_t$  at two loop level:

$$\begin{aligned} \frac{dy_t}{dt} &= \frac{y_t}{2(4\pi)^2} \left( \left( \frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 - 6y_t^2 - 3\kappa_{10}^2 \right) \right. \\ &- \frac{1}{(4\pi)^2} \left( \frac{3913}{450} g_1^4 + \frac{33}{2} g_2^4 + \frac{128}{9} g_3^4 + g_1^2 g_2^2 + \frac{136}{45} g_1^2 g_3^2 + 8g_2^2 g_3^2 + \left( \frac{2}{5} g_1^2 + 6g_2^2 \right) y_t^2 \right. \\ &+ \left. \left. \left( \frac{4}{5} g_1^2 + 16g_3^2 \right) (y_t^2 + \kappa_{10}^2) - 9(y_t^4 + \kappa_{10}^4) - 9y_t^2 (y_t^2 + \kappa_{10}^2) - 4y_t^4 \right) \right). \end{aligned} \quad (11)$$

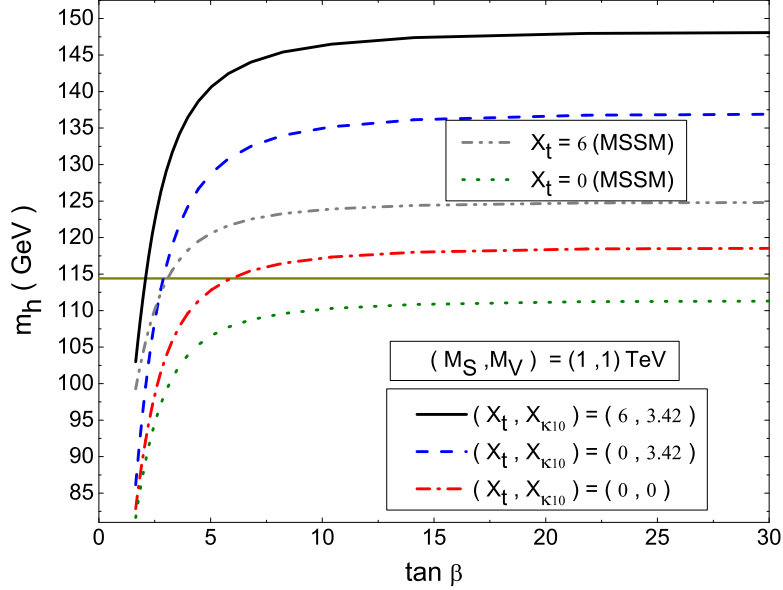


Figure 3: Upper bounds for the lightest CP-even Higgs boson mass vs  $\tan \beta$  for different maximal and minimal values of  $X_t$ ,  $X_{\kappa_{10}}$ , with  $M_S = 1$  TeV,  $M_V = 1$  TeV and  $M_t = 172.6$  GeV. Dotted line corresponds to MSSM ( $X_t = 0$ ). Dashed-double dotted line describes MSSM with ( $X_t = 6$ ). Dashed-dotted curve is for MSSM +  $10 + \overline{10}$ .  $\kappa_{10} \approx 2$  at  $M_G$ . Dashed line shows Higgs mass with  $X_{\kappa_{10}} = 2.95$  and  $X_t = 0$ . Solid line corresponds to  $X_{\kappa_{10}} = 2.95$  and  $X_t = 6$ .

The additional vectorlike matter fields also modify the RGE's for the MSSM gauge couplings and the corresponding beta functions can be found in [14].

In our calculation, the weak scale ( $M_Z$ ) value of the gauge and top Yukawa couplings are evolved to the scale  $M_G$  via the RGE's in  $\overline{DR}$  regularization scheme, where the scale  $M_G$  is defined to be one where  $g_2 = g_1$ . We do not enforce an exact unification of the strong coupling  $g_3 = g_2 = g_1$  at scale  $M_G$ , since a few percent deviation from the unification condition can be assigned to unknown GUT scale threshold corrections. At the scale  $M_G$  we impose  $\kappa_{10} \approx 2$ , in order to obtain the maximal value for  $\kappa_{10}$  at low scale, which is consistent with the  $T$  parameter constraints. In this case we can generate, according to Eq. (9), the maximal plausible values for Higgs mass. Our goal is to achieve the maximal value for the CP-even Higgs mass and, as we show in Eq. (27), the Higgs mass is proportional to  $\kappa_{10}^4$ . The coupling  $\kappa_{10}$ , along with the gauge and top Yukawa couplings, are evolved back to  $M_Z$ . In the evolution of couplings, for the SUSY threshold correction we follow the effective SUSY scale approach, according to which all SUSY particles are assumed to lie at an effective scale [15]. Below  $M_{SUSY}$  we employ the non-SUSY RGEs. All of the couplings are iteratively run between  $M_Z$  and  $M_G$  using two loop RGEs for both the Yukawa and gauge couplings until a stable solution is obtained. Note that  $M_S$  and  $M_{SUSY}$  are distinct parameters. As pointed out in ref. [15], one can have a different set of values for stop squark masses for a given effective  $M_{SUSY}$  and so correspondingly one considers different values of  $M_S$  for  $M_{SUSY} = 200$  GeV.

Requiring  $\delta T < 0.1$  with  $10 + \overline{10}$  masses at  $M_V = 1$  TeV, we find that  $\kappa_{10}(M_V) < 1.142$  at

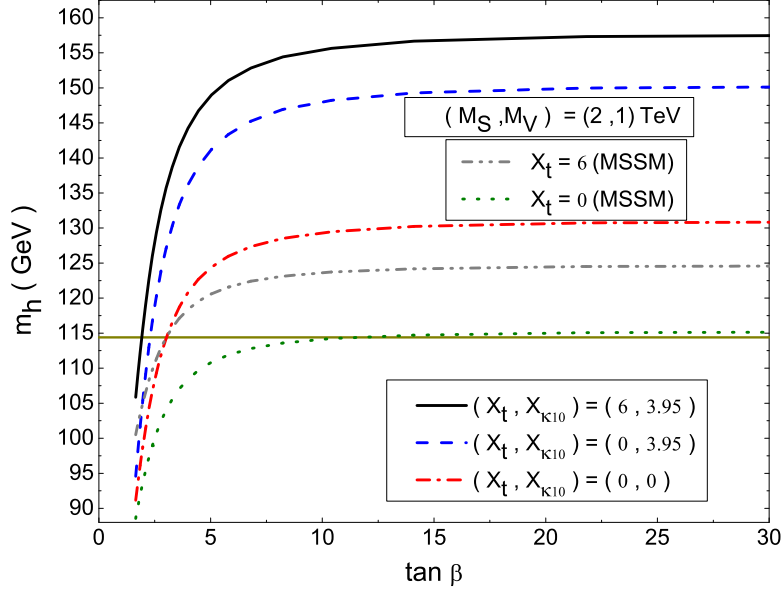


Figure 4: Upper bounds for the lightest CP-even Higgs boson mass vs  $\tan \beta$  for different maximal and minimal values of  $X_t$ ,  $X_{\kappa_{10}}$ , with  $M_S = 2$  TeV,  $M_V = 1$  TeV and  $M_t = 172.6$  GeV. Dotted line corresponds to MSSM ( $X_t = 0$ ). Dashed-double dotted line describes MSSM with ( $X_t = 6$ ). Dashed-dotted curve is for MSSM +10+ $\overline{10}$ .  $\kappa_{10} \approx 2$  at  $M_G$ . Dashed line shows Higgs mass with  $X_{\kappa_{10}} = 3.95$  and  $X_t = 0$ . Solid line corresponds to  $X_{\kappa_{10}} = 3.95$  and  $X_t = 6$ .

( $M_V$ ) scale using the formula from ref. [9]. The corresponding  $\kappa_{10}$  at GUT scale in this case is  $\kappa_{10}(M_G) \approx 2$ . We find that the  $S$ -parameter constraint is automatically satisfied once the  $T$ -parameter constraint is met.

We can see from Eqs. (4) – (8) that to maximize the CP-even Higgs boson mass we should not only take the maximal allowed value for  $\kappa_{10}$ , we also need to have the maximal values for the parameters  $X_t$  and  $X_{\kappa_{10}}$ . According to Eq. (5) we find that  $X_{\kappa_{10}} = 2.95$ , with  $M_S = 500$  GeV and  $M_V = 1$  TeV. The value for  $X_{\kappa_{10}}$  increases ( $X_{\kappa_{10}} = 3.42$ ) if we consider  $M_S = 1$  TeV and  $M_V = 1$  TeV, while  $X_{\kappa_{10}} = 3.95$  for  $M_S = 2$  TeV and  $M_V = 1$  TeV.

We find that  $M_V = 1$  TeV is somehow the optimum value for the vectorlike particle mass, especially because the  $T$  parameter constraint almost disappears for this value of  $M_V$ . On the other hand Eq. (22) does not allow very low values for  $M_S$  if significant corrections are to be realized. This is the reason why we choose  $M_S = 0.5, 1$  and  $2$  TeV for our analysis.

In Figure 2 we present the upper bounds for the CP-even Higgs boson mass vs  $\tan \beta$  with different maximal or minimal values of  $X_t$ ,  $X_{\kappa_{10}}$  when  $M_S = 500$  GeV and  $M_V = 1$  TeV, and we compare to the MSSM case. We take at scale  $M_G$ ,  $\kappa_{10} \approx 2$  to obtain the maximal effect for the lightest Higgs boson mass. As we see from Figure 2, for this choice of parameters the maximal values for Higgs mass is 141 GeV. In Figure 3 we present the results for the case in which the mass for vectorlike matter is  $M_V = 1$  TeV and  $M_S = 1$  TeV too. In this case the CP-even higgs mass can be as large as 148 GeV. Finally in Figure 4 we consider  $M_V = 1$  TeV and  $M_S = 2$  TeV case and obtain the maximal value of 158 GeV for the Higgs mass.



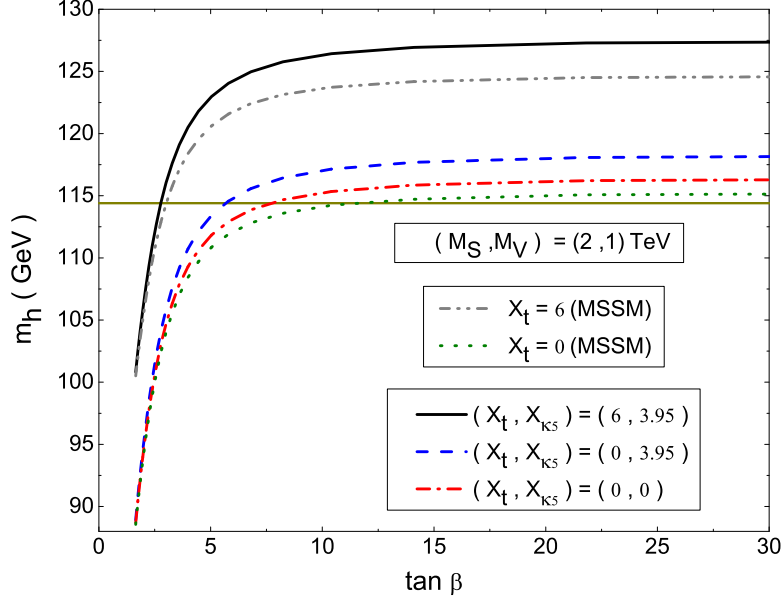


Figure 5: Upper bounds for the lightest CP-even Higgs boson mass vs  $\tan \beta$  for different maximal and minimal values of  $X_t$ ,  $X_{\kappa_5}$ , with  $M_S = 2$  TeV,  $M_V = 1$  TeV and  $M_t = 172.6$  GeV. Dotted line corresponds to MSSM with  $(X_t = 0)$ . Dashed-double dotted line describes MSSM with  $(X_t = 6)$ . Dashed-dotted curve is for MSSM  $+5 + \bar{5}$ .  $\kappa_5 \approx 2$  at  $M_G$ . Dashed line shows Higgs mass with  $X_{\kappa_5} = 3.95$  and  $X_t = 0$ . Solid line corresponds to  $X_{\kappa_5} = 3.95$  and  $X_t = 6$ .

### 3.2 MSSM $+ 5 + \bar{5}$

In this subsection we consider the case in which at the TeV scale we have extra matter which belongs to the 5 dimensional representation of  $SU(5)$ . This decomposes under the MSSM gauge symmetry as follows:

$$5 + \bar{5} = L_5 \left( 1, 2, \frac{1}{2} \right) + \bar{L}_5 \left( 1, 2, -\frac{1}{2} \right) + D_5 \left( \bar{3}, 1, \frac{1}{3} \right) + \bar{D}_5 \left( 3, 1, -\frac{1}{3} \right). \quad (12)$$

Our goal is to generate new trilinear couplings of this extra matter with the MSSM Higgs fields. The  $5 + \bar{5}$  itself cannot generate this kind of coupling. However, if we introduce an MSSM singlet  $S$ , then Yukawa couplings of the form (in  $SU(5)$  notation)  $\bar{5} \cdot S \cdot H_u$  and  $5 \cdot S \cdot H_d$  are permitted. In this case the MSSM superpotential has the following additional contribution

$$W = \kappa_5 L_5 S H_u + \kappa'_5 \bar{L}_5 S H_d + M_V (S \bar{S} + \bar{L}_5 L_5 + \bar{D}_5 D_5). \quad (13)$$

We take  $\kappa_5 \gg \kappa'_5$ , for the same reason mentioned in the previous section. We also assume that there is an additional symmetry which forbids the mixing of the vectorlike particle with the MSSM matter fields. With this assumption the singlet field  $S$  cannot be identified with the right handed neutrino.

Using the effective potential approach we calculate the additional contribution to the CP

even Higgs mass at one loop level. [A similar calculation was done in ref. [11].]

$$[m_h^2]_5 = -M_Z^2 \cos^2 2\beta \left( \frac{1}{8\pi^2} \kappa_5^2 t_V \right) + \frac{1}{4\pi^2} \kappa_5^4 v^2 \sin^2 \beta \left[ t_V + \frac{1}{2} X_{\kappa_5} \right], \quad (14)$$

where we have assumed  $M_V \gg M_D$  and

$$X_{\kappa_5} = \frac{4\tilde{A}_{\kappa_5}^2 (3M_S^2 + 2M_V^2) - \tilde{A}_{\kappa_5}^4 - 8M_S^2 M_V^2 - 10M_S^4}{6(M_S^2 + M_V^2)^2} \quad (15)$$

and

$$t_V = \log \left( \frac{M_S^2 + M_V^2}{M_V^2} \right). \quad (16)$$

Here  $\tilde{A}_{\kappa_5} = A_{\kappa_5} - \mu \cot \beta$ ,  $A_{\kappa_5}$  is the  $L_5 - S$  soft mixing parameter and  $\mu$  is the MSSM Higgs bilinear mixing term.

The RGE for  $\kappa_5$  has the following form

$$\begin{aligned} \frac{d\kappa_5}{dt} = & \frac{\kappa_5}{2(4\pi)^2} \left( \left( 3g_2^2 + \frac{3}{5}g_1^2 - 4\kappa_5^2 - 3y_t^2 \right) \right. \\ & - \frac{1}{(4\pi)^2} \left( \frac{237}{50}g_1^4 + \frac{21}{2}g_2^4 + \frac{9}{5}g_1^2 g_2^2 + \left( \frac{6}{5}g_1^2 + 6g_2^2 \right) \kappa_5^2 \right. \\ & \left. \left. + \left( \frac{4}{5}g_1^2 + 16g_3^2 \right) y_t^2 - 3(3y_t^4 + \kappa_5^4) - 3\kappa_5^2(3y_t^2 + \kappa_5^2) - 4\kappa_5^4 \right) \right). \end{aligned} \quad (17)$$

Because the new matter fields couple to  $H_u$  (see Eq. (13)), there are additional contribution to the RGE for  $y_t$  which to two-loop level is given by

$$\begin{aligned} \frac{dy_t}{dt} = & \frac{y_t}{2(4\pi)^2} \left( \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 - 6y_t^2 - \kappa_5^2 \right) \right. \\ & - \frac{1}{(4\pi)^2} \left( \frac{3133}{450}g_1^4 + \frac{21}{2}g_2^4 + \frac{32}{9}g_3^4 + g_1^2 g_2^2 + \frac{136}{45}g_1^2 g_3^2 + 8g_2^2 g_3^2 + \left( \frac{2}{5}g_1^2 + 6g_2^2 \right) y_t^2 \right. \\ & \left. \left. + \left( \frac{4}{5}g_1^2 + 16g_3^2 \right) y_t^2 - 3(3y_t^4 + \kappa_5^4) - 3y_t^2(3y_t^2 + \kappa_5^2) - 4y_t^4 \right) \right). \end{aligned} \quad (18)$$

We can see from Eq. (17) that  $\kappa_5$  cannot be as large at  $M_Z$  scale as  $\kappa_{10}$  was. The reason for this is that in the RGE for  $\kappa_5$ , in contrast to the case for  $\kappa_{10}$ , the strong gauge coupling does not participate at one loop level. Because of this we find that  $\kappa_5(M_Z) = 0.74$  for  $M_S = 2$  TeV and  $M_V = 1$  TeV.

In Figure 5 we present the upper bounds for the CP-even Higgs boson mass vs  $\tan \beta$  with different maximal or minimal values of  $X_t$ ,  $X_{\kappa_5}$  with  $M_S = 2$  TeV and  $M_V = 1$  TeV and which we compare with the MSSM case. We take  $\kappa_5 \approx 2$  at scale  $M_G$  as before. For this choice of parameters the maximal value for the Higgs mass is 127.5 GeV.

### 3.3 MSSM + 5 + $\bar{5}$ + 10 + $\bar{10}$

In this section we will consider extra vectorlike matter belonging to the representation  $5 + \bar{5} + 10 + \bar{10}$  of SU(5). There are two choices to consider here, namely with or without two SM singlet fields. This does not affect the perturbativity condition, but the presence of the singlets suggests an underlying SO(10) gauge symmetry.

*Case I.* Without the singlet the MSSM superpotential acquires the following additional contribution

$$\begin{aligned} W = & \kappa_1 Q_{10} U_{10} H_u + \kappa_2 \bar{Q}_{10} \bar{D}_5 H_u + \kappa_3 \bar{Q}_{10} \bar{U}_{10} H_d + \kappa_4 Q_{10} D_5 H_d \\ & + M_V (\bar{Q}_{10} Q_{10} + \bar{U}_{10} U_{10} + \bar{E}_{10} E_{10} + \bar{L}_5 L_5 + \bar{D}_5 D_5). \end{aligned} \quad (19)$$

The new interaction yields the following additional contribution to the MSSM CP-even Higgs boson mass

$$\begin{aligned} [m_h^2]_1 = & - M_Z^2 \cos^2 2\beta \left( \frac{3}{8\pi^2} \kappa_1^2 t_V \right) + \frac{3}{4\pi^2} \kappa_1^4 v^2 \sin^2 \beta \left[ t_V + \frac{1}{2} X_{\kappa_1} \right], \\ & - M_Z^2 \cos^2 2\beta \left( \frac{3}{8\pi^2} \kappa_2^2 t_V \right) + \frac{3}{4\pi^2} \kappa_2^4 v^2 \sin^2 \beta \left[ t_V + \frac{1}{2} X_{\kappa_2} \right], \end{aligned} \quad (20)$$

where we have assumed  $M_V \gg M_D$ , and defined

$$X_{\kappa_i} = \frac{4\tilde{A}_{\kappa_i}^2 (3M_S^2 + 2M_V^2) - \tilde{A}_{\kappa_i}^4 - 8M_S^2 M_V^2 - 10M_S^4}{6(M_S^2 + M_V^2)^2}, \quad (21)$$

and

$$t_V = \log \left( \frac{M_S^2 + M_V^2}{M_V^2} \right), \quad (22)$$

where  $i = 1, 2$ ,  $\tilde{A}_{\kappa_i} = A_{\kappa_i} - \mu \cot \beta$  and  $A_{\kappa_i}$  is the soft mixing parameter.

In this case the lightest CP-even Higgs mass is

$$m_h^2 = [m_h^2]_{MSSM} + [m_h^2]_1 \quad (23)$$

where the expression for  $[m_h^2]_{MSSM}$  is given in Eq. (7)

The RGEs for  $\kappa_1$  and  $\kappa_2$  are given to one-loop order by

$$\begin{aligned} \frac{d\kappa_1}{dt} &= \frac{\kappa_1}{2(4\pi)^2} \left( \frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 - 6\kappa_1^2 - 3\kappa_2^2 - 3y_t^2 \right), \\ \frac{d\kappa_2}{dt} &= \frac{\kappa_2}{2(4\pi)^2} \left( \frac{16}{3} g_3^2 + 3g_2^2 + \frac{7}{15} g_1^2 - 6\kappa_2^2 - 3\kappa_1^2 - 3y_t^2 \right). \end{aligned} \quad (24)$$

Because the new matter couples to  $H_u$  (see Eq. (19)) there is an additional contribution to the RGE for  $y_t$  at one-loop level:

$$\frac{dy_t}{dt} = \left[ \frac{dy_t}{dt} \right]_{MSSM} - \frac{3}{2(4\pi)^2} y_t \kappa_1^2 - \frac{3}{2(4\pi)^2} y_t \kappa_2^2. \quad (25)$$

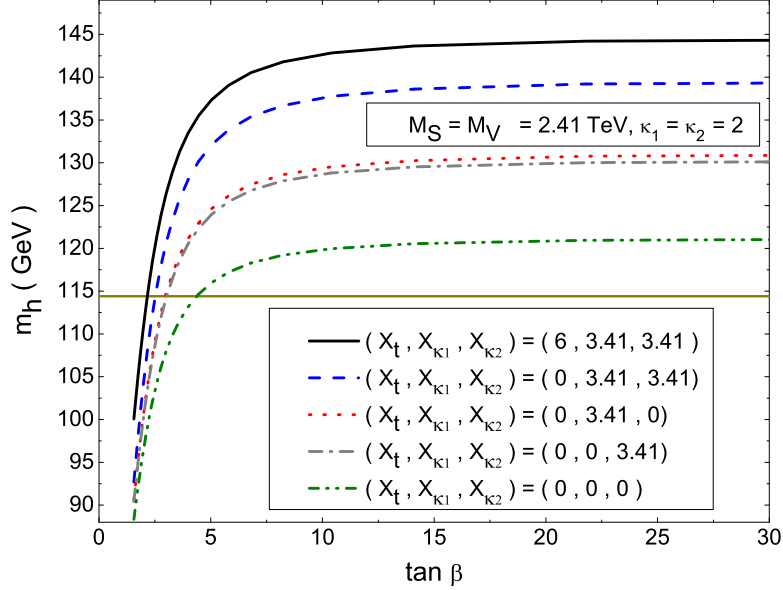


Figure 6: Upper bounds for the lightest CP-even Higgs boson mass vs  $\tan \beta$  for different maximal and minimal values of  $X_t$ ,  $X_{\kappa_1}$ ,  $X_{\kappa_2}$ , with  $M_S = M_V = 2.41$  TeV, and  $M_t = 172.6$  GeV. Dashed-double dotted curve describes the logarithmical correction in the model. Dotted curve corresponds to the maximum value of  $\kappa_1$  or  $\kappa_2$ . Dashed curve corresponds to the maximum values of  $\kappa_1$  and  $\kappa_2$ . Solid curve corresponds to the case when all corrections are taken to be maximum.

In Figure 6 we present the upper bounds for the CP-even Higgs boson mass vs  $\tan \beta$  for different maximal or minimal values of  $X_t$ ,  $X_{\kappa_1}$ ,  $X_{\kappa_2}$ , with  $M_S = M_V = 2.41$  TeV, and compare it with the MSSM case. We take  $\kappa_i \approx 2$  at  $M_G$  as before. For the given choice of parameters the maximal value of the Higgs mass is 144.5 GeV.

*Case II.* Next we consider the case when at low scale we have vectorlike particles in  $(16 + \overline{16})$  dimensional representation of  $SO(10)$ . The MSSM superpotential for this case acquires the following additional contribution:

$$\begin{aligned}
W = & \kappa_1 Q_{10} U_{10} H_u + \kappa_2 \overline{Q}_{10} \overline{D}_5 H_u + \kappa_3 L_5 S H_u + \kappa_4 Q_{10} D_5 H_d + \kappa_5 \overline{Q}_{10} \overline{U}_{10} H_d + \kappa_6 \overline{L}_5 H_d \overline{S} \\
& + M_V (\overline{Q}_{10} Q_{10} + \overline{U}_{10} U_{10} + \overline{E}_{10} E_{10} + \overline{L}_5 L_5 + \overline{D}_5 D_5 + \overline{S} S). \quad (26)
\end{aligned}$$

The new interactions provide the following additional contribution to the MSSM CP-even Higgs boson mass

$$\begin{aligned}
[m_h^2]_2 = & - M_Z^2 \cos^2 2\beta \left( \frac{3}{8\pi^2} \kappa_1^2 t_V \right) + \frac{3}{4\pi^2} \kappa_1^4 v^2 \sin^2 \beta \left[ t_V + \frac{1}{2} X_{\kappa_1} \right], \\
& - M_Z^2 \cos^2 2\beta \left( \frac{3}{8\pi^2} \kappa_2^2 t_V \right) + \frac{3}{4\pi^2} \kappa_2^4 v^2 \sin^2 \beta \left[ t_V + \frac{1}{2} X_{\kappa_2} \right], \\
& - M_Z^2 \cos^2 2\beta \left( \frac{1}{8\pi^2} \kappa_3^2 t_V \right) + \frac{1}{4\pi^2} \kappa_3^4 v^2 \sin^2 \beta \left[ t_V + \frac{1}{2} X_{\kappa_3} \right], \quad (27)
\end{aligned}$$

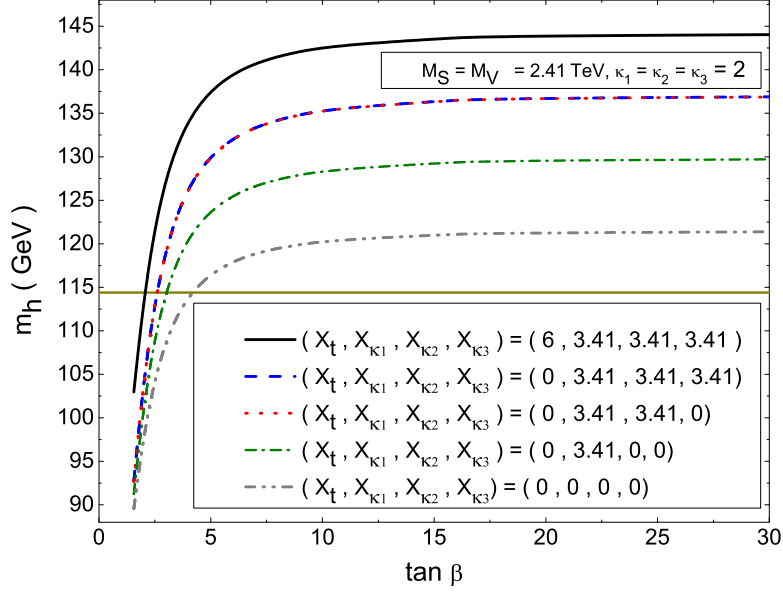


Figure 7: Upper bounds for the lightest CP-even Higgs boson mass vs  $\tan \beta$  for different maximal and minimal values of  $X_t$ ,  $X_{\kappa_1}$ ,  $X_{\kappa_2}$ ,  $X_{\kappa_3}$ , with  $M_S = M_V = 2.41$  TeV and  $M_t = 172.6$  GeV.

where we have assumed  $M_V \gg M_D$ , and defined

$$X_{\kappa_i} = \frac{4\tilde{A}_{\kappa_i}^2 (3M_S^2 + 2M_V^2) - \tilde{A}_{\kappa_i}^4 - 8M_S^2 M_V^2 - 10M_S^4}{6(M_S^2 + M_V^2)^2}, \quad (28)$$

and

$$t_V = \log \left( \frac{M_S^2 + M_V^2}{M_V^2} \right). \quad (29)$$

Here  $i = 1, 2, 3$  and  $\tilde{A}_{\kappa_i} = A_{\kappa_i} - \mu \cot \beta$ . The lightest CP-even Higgs mass is

$$m_h^2 = [m_h^2]_{MSSM} + [m_h^2]_2, \quad (30)$$

where the expression for  $[m_h^2]_{MSSM}$  is given in Eq. (7).

The RGEs for  $\kappa_i$  are given by:

$$\begin{aligned} \frac{d\kappa_1}{dt} &= \frac{\kappa_1}{2(4\pi)^2} \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 - 6\kappa_1^2 - 3\kappa_2^2 - \kappa_3^2 - 3y_t^2 \right), \\ \frac{d\kappa_2}{dt} &= \frac{\kappa_2}{2(4\pi)^2} \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2 - 6\kappa_2^2 - 3\kappa_1^2 - \kappa_3^2 - 3y_t^2 \right), \\ \frac{d\kappa_3}{dt} &= \frac{\kappa_3}{2(4\pi)^2} \left( 3g_2^2 + \frac{3}{5}g_1^2 - 4\kappa_3^2 - 3\kappa_1^2 - 3\kappa_2^2 - 3y_t^2 \right). \end{aligned} \quad (31)$$

The RGE for  $y_t$  is modified as follows:

$$\frac{dy_t}{dt} = \left[ \frac{dy_t}{dt} \right]_{MSSM} - \frac{3}{2(4\pi)^2} y_t \kappa_1^2 - \frac{3}{2(4\pi)^2} y_t \kappa_2^2 - \frac{1}{2(4\pi)^2} y_t \kappa_3^2. \quad (32)$$

In Figure 7 we present the upper bounds for the CP-even Higgs boson mass vs  $\tan \beta$  for different maximal minimal values of  $X_t$ ,  $X_{\kappa_1}$ ,  $X_{\kappa_2}$ ,  $X_{\kappa_3}$ , with  $M_S = M_V = 2.41$  TeV, and compare it with the MSSM case. We take  $\kappa_i \approx 2$  at  $M_G$  as before. For the given choice of parameters the maximal value of the Higgs mass is 143.9 GeV. It is worth noting that the resultant Higgs mass bound for *Case II* more or less coincides with what we found for *Case I* (see Figure 6). This stems from the fact that the contribution at ‘low’ scale from the new coupling  $\kappa_3$  is small due to the absence of the strong coupling (see Eq. (31)).

## 4 Little Hierarchy problem

### 4.1 MSSM

As discussed in Section 3, in the MSSM at tree level the lightest CP even Higgs boson mass is bounded from above by the mass of the  $Z$  boson

$$m_h^2 < M_Z^2 \cos 2\beta. \quad (33)$$

It requires large radiative corrections in order to push the lightest Higgs mass above the LEP2 limit. We can see that there are two kind of correction (see Eq. (7)), one proportional to  $m_t^4 \log(M_S^2/m_t^2)$ , where  $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ , and the second proportional to  $A_t$ . As we can see from Figure 8, if the Higgs mass turns out to be much heavier then 114.4 GeV, we need not only a large trilinear soft SUSY breaking  $A_t$  term but also heavy stop quark masses.

On the other hand, the mass of the  $Z$  boson ( $M_Z \simeq 91$  GeV) is given from the minimization of the scalar potential as (for  $\tan \beta \gtrsim 5$ )

$$\frac{1}{2} M_Z^2 \simeq -\mu^2 - m_{H_u}^2, \quad (34)$$

and the radiative correction to the soft scalar mass squared for  $H_u$  is proportional to top squark masses

$$\Delta m_{H_u}^2(M_Z) = -\frac{3y_t^2(M_Z)}{4\pi^2} M_S^2 \ln \frac{\Lambda}{M_S}, \quad (35)$$

where  $\Lambda$  is a more fundamental scale, such as  $M_G$ .

Thus, in the MSSM one needs to have heavy top squarks to generate the lightest Higgs mass above the LEP2 bound, while on the other hand from Eqs. (34) and (35) we see that some fine tuning is needed to get the correct  $Z$  boson mass. This is known as the *little hierarchy problem*. To see how much fine tuning is needed to satisfy Eq. (34) we performed semi-analytic calculation for the MSSM sparticle spectra with the following boundary conditions

$$\{\alpha_G, M_G, y_t(M_G)\} = \{1/24.32, 2.0 \times 10^{16}, 0.512\}. \quad (36)$$

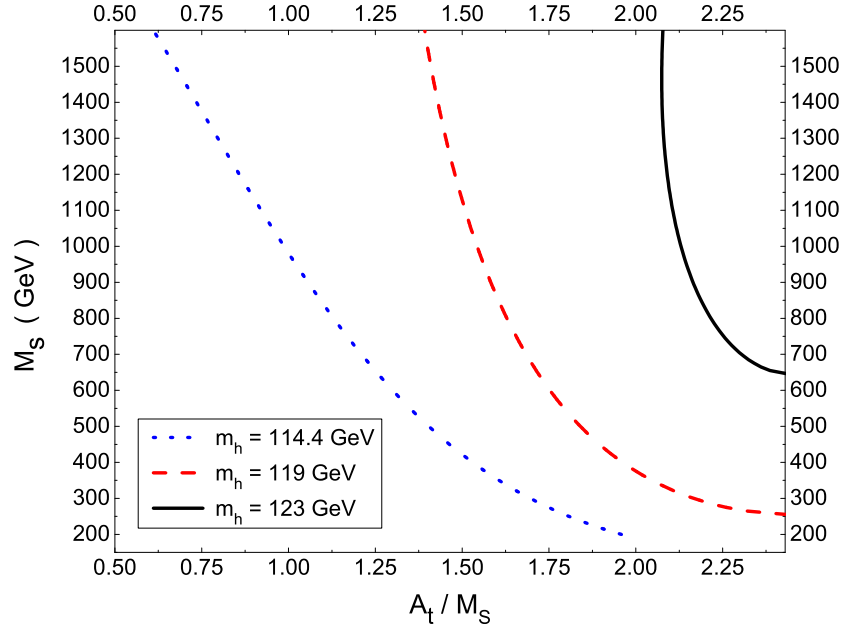


Figure 8:  $M_S$  versus  $A_t/M_S$  for different values of CP-even Higgs mass in MSSM. Dotted, dashed and solid line corresponds to Higgs mass  $m_h=114.4, 119$  and  $123$  GeV respectively and  $\tan\beta = 10$ .

We express the MSSM sparticle masses at the scale  $M_Z$  in terms of the GUT/Planck scale fundamental parameters  $(m_0, m_{1/2}, A_{t_0})$  and the Higgs bi-linear mixing term  $\mu$ , by integrating the one loop renormalization group equations [17]. For example, the gaugino masses at  $M_Z$  scale are

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} = \{0.412, 0.822, 2.844\}m_{1/2}. \quad (37)$$

The scalar particle masses,  $A_t$  and  $\mu$  at the  $M_Z$  scale are given by

$$-m_{H_u}^2(M_Z) = 2.72m_{1/2}^2 + 0.091m_0^2 + 0.1A_{t_0}^2 - 0.43m_{1/2}A_{t_0} \quad (38)$$

$$m_{Q_t}^2(M_Z) = 5.71m_{1/2}^2 + 0.64m_0^2 - 0.033A_{t_0}^2 + 0.15m_{1/2}A_{t_0} \quad (39)$$

$$m_{U_t}^2(M_Z) = 4.2m_{1/2}^2 + 0.27m_0^2 - 0.07A_{t_0}^2 + 0.29m_{1/2}A_{t_0} \quad (40)$$

$$A_t(M_Z) = -2.3m_{1/2} + 0.27A_{t_0} \quad (41)$$

$$m_{Q_{1,2}}^2(M_Z) = 6.79m_{1/2}^2 + m_0^2 \quad (42)$$

$$m_{U_{1,2}}^2(M_Z) = 6.37m_{1/2}^2 + m_0^2 \quad (43)$$

$$m_{D_{1,2,3}}^2(M_Z) = 6.32m_{1/2}^2 + m_0^2 \quad (44)$$

$$m_{L_{1,2,3}}^2(M_Z) = 0.52m_{1/2}^2 + m_0^2 \quad (45)$$

$$m_{E_{1,2,3}}^2(M_Z) = 0.15m_{1/2}^2 + m_0^2 \quad (46)$$

$$\mu^2(M_Z) = 1.02\mu_0^2. \quad (47)$$

where the subscript 1,2,3 are families indices and  $\mu_0$  is the value of  $\mu$  at  $M_G$ .

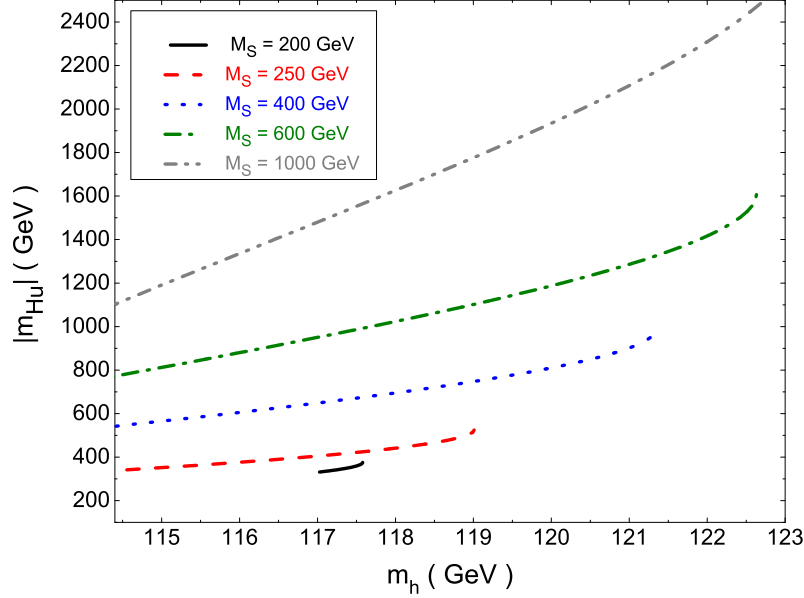


Figure 9:  $|m_{H_U}|$  versus CP-even Higgs mass in the MSSM for different values of  $M_S$ . Solid, dashed, dotted, dashed-dotted and dashed-dotted-dotted curve corresponds  $M_S = 200$  GeV, 250 GeV, 400 GeV, 600 GeV and 1000 GeV respectively. In each case  $A_t$  varies in the interval  $0 < |A_t/M_S| < \sqrt{6}$ .

Using Eq. (34) we can also express the dominant contribution to  $Z$  boson mass in terms of fundamental parameters:

$$M_Z^2 \simeq -2.04\mu^2 + 5.44m_{1/2}^2 + 0.183m_0^2 + 0.2A_{t_0}^2 - 0.87m_{1/2}A_{t_0}. \quad (48)$$

The magnitude of  $|m_{H_U}|$  shows how much fine tuning is needed to satisfy the minimization condition in the MSSM, (see Eq. (34)). We present in Figure 9  $|m_{H_U}|$  versus the CP-even Higgs mass for different values of  $M_S$ . In each case  $A_t$  varies in the interval  $0 < |A_t/M_S| < \sqrt{6}$ . This is the reason why we find different values for the Higgs masses for different choice of  $M_S$ .

We find that the new Yukawa couplings of  $H_u$  to the vectorlike matter, which helps in raising  $m_h$ , also has the effect of raising the soft Higgs mass parameter  $m_{H_u}^2$ , which tends to exacerbate the little hierarchy problem. However, when the new Yukawa couplings of  $H_u$  are relatively small, the little hierarchy problem improves relative to the MSSM, since the cumulative effect of the top Yukawa coupling  $y_t$  on the running of  $m_{H_u}^2$  becomes smaller than in MSSM. This comes about since the value of  $y_t$  is smaller at the scale  $M_G$  compared to MSSM for certain values of the new Yukawa coupling. This result is displayed in Figure 10 for the MSSM+10+ $\overline{10}$  case. There is also contribution from radiative correction involving the gluon and gluino, since, as we show in Figure 1, introducing new vectorlike matter at low scale slows the running of strong coupling compared to the MSSM case. As a result the cumulative effect of the strong interaction to the running of colored particle masses is smaller than in MSSM. These two effects, as we show in the next two sections, enable us to improve the little hierarchy problem compared to the MSSM.



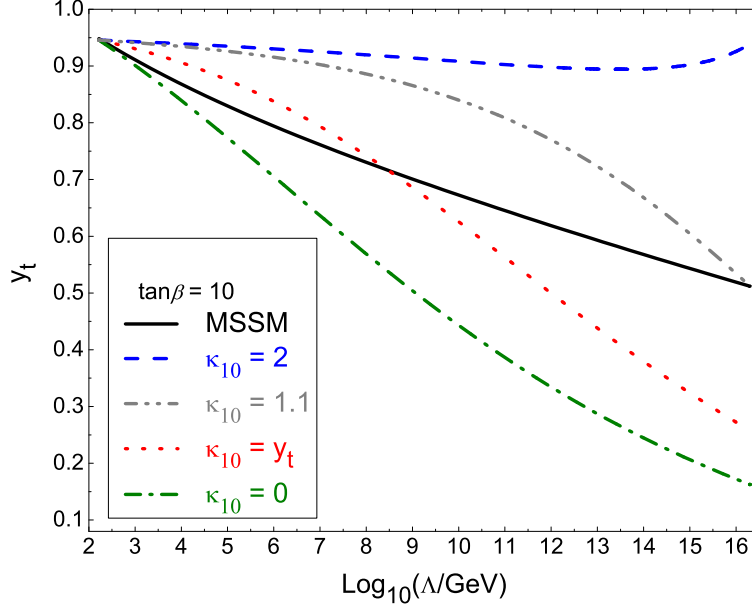


Figure 10: Top Yukawa  $y_t$  coupling versus  $\text{Log}_{10}(\Lambda/\text{GeV})$  for  $\tan\beta = 10$ . Dashed, dotted, dashed-dotted and dashed-dotted-dotted lines correspond to MSSM+10 +  $\bar{10}$  with  $\kappa_{10} = 2, y_t, 0, 1.1$  respectively. Solid line correspond to the MSSM case.

## 4.2 MSSM+10 + $\bar{10}$

Next let us consider how the little hierarchy problem can be improved in the MSSM+10 +  $\bar{10}$  case. We will consider two extreme values for the coupling  $\kappa_{10}$ , namely  $\kappa_{10}(M_G) = 2$  and  $\kappa_{10}(M_G) = 0$  to show how much little hierarchy has changed for this case.

*Case I.* Using the boundary conditions

$$\{\alpha_G, M_G, y_t(M_G), \kappa_{10}(M_G)\} = \{1/8.55, 2.0 \times 10^{16}, 0.94, 2\}. \quad (49)$$

and RGEs from Appendix A, we obtain the particle spectrum. For the gaugino masses,

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} = \{0.145, 0.289, 1\}m_{1/2}, \quad (50)$$

while the MSSM scalar masses along with  $\mu^2$ ,  $A_t$  and  $A_{\kappa_{10}}$  are given by

$$\begin{aligned} -m_{H_u}^2(M_Z) &= 3.85m_{1/2}^2 + 0.95m_0^2 + 0.04A_{t_0}^2 + 0.012A_{\kappa_{10_0}}^2 \\ &\quad - 0.12m_{1/2}A_{t_0} + 0.06m_{1/2}A_{\kappa_{10_0}} - 0.043A_{t_0}A_{\kappa_{10_0}} \end{aligned} \quad (51)$$

$$\begin{aligned} m_{Q_t}^2(M_Z) &= 2.98m_{1/2}^2 + 0.73m_0^2 - 0.031A_{t_0}^2 - 0.002A_{\kappa_{10_0}}^2 \\ &\quad + 0.11m_{1/2}A_{t_0} - 0.071m_{1/2}A_{\kappa_{10_0}} + 0.026A_{t_0}A_{\kappa_{10_0}} \end{aligned} \quad (52)$$

$$\begin{aligned} m_{U_t}^2(M_Z) &= 2.04m_{1/2}^2 + 0.45m_0^2 - 0.062A_{t_0}^2 - 0.003A_{\kappa_{10_0}}^2 \\ &\quad + 0.23m_{1/2}A_{t_0} - 0.14m_{1/2}A_{\kappa_{10_0}} + 0.052A_{t_0}A_{\kappa_{10_0}} \end{aligned} \quad (53)$$

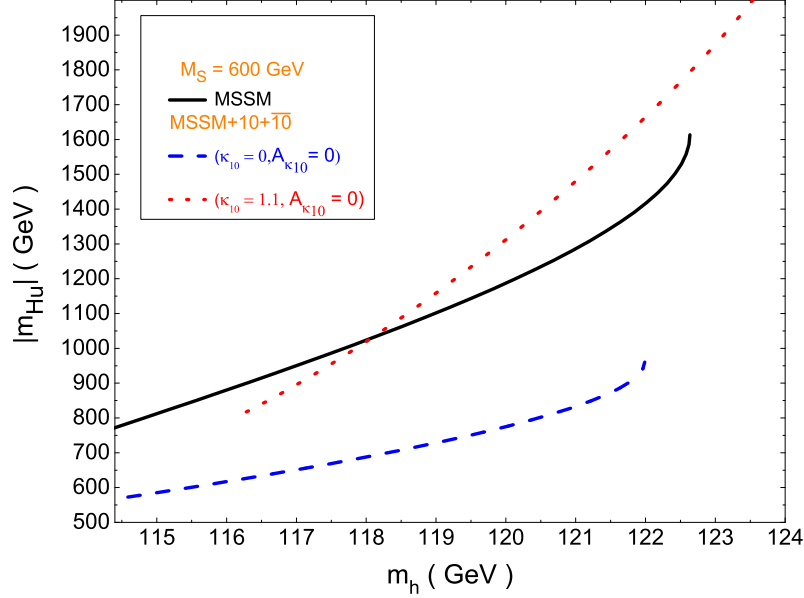


Figure 11:  $|m_{H_U}|$  versus the CP even Higgs mass  $m_h$ , with  $M_S = 600$  GeV. Solid line correspond to the MSSM case. Dashed and dotted curve correspond to MSSM+10 +  $\overline{10}$ , with  $\kappa_{10} = 1.1$  and  $\kappa_{10} = 0$  at  $M_G$ .

$$A_t(M_Z) = -1.02m_{1/2} + 0.2A_{t_0} - 0.13A_{\kappa_{10_0}} \quad (54)$$

$$A_{\kappa_{10}}(M_Z) = -0.71m_{1/2} - 0.093A_{t_0} + 0.065A_{\kappa_{10_0}} \quad (55)$$

$$m_{Q_{1,2}}^2(M_Z) = 3.63m_{1/2}^2 + m_0^2 \quad (56)$$

$$m_{U_{1,2}}^2(M_Z) = 3.33m_{1/2}^2 + m_0^2 \quad (57)$$

$$\mu^2(M_Z) = 0.105\mu_0^2 \quad (58)$$

$$m_{D_{1,2,3}}^2(M_Z) = m_0^2 + 3.29m_{1/2}^2 \quad (59)$$

$$m_{L_{1,2,3}}^2(M_Z) = m_0^2 + 0.37m_{1/2}^2 \quad (60)$$

$$m_{E_{1,2,3}}^2(M_Z) = m_0^2 + 0.122m_{1/2}^2. \quad (61)$$

The spectrum for the vectorlike matter is given as

$$m_{Q_{10}}^2(M_Z) = 2.86m_{1/2}^2 + 0.62m_0^2 + 0.017A_{t_0}^2 - 0.003A_{\kappa_{10_0}}^2 - 0.073m_{1/2}A_{t_0} + 0.051m_{1/2}A_{\kappa_{10_0}} - 0.011A_{t_0}A_{\kappa_{10_0}} \quad (62)$$

$$m_{U_{10}}^2(M_Z) = 1.81m_{1/2}^2 + 0.25m_0^2 + 0.035A_{t_0}^2 - 0.005A_{\kappa_{10_0}}^2 - 0.15m_{1/2}A_{t_0} + 0.1m_{1/2}A_{\kappa_{10_0}} - 0.023A_{t_0}A_{\kappa_{10_0}} \quad (63)$$

$$m_{E_{10}}^2(M_Z) = m_0^2 + 0.122m_{1/2}^2. \quad (64)$$

For this case the dominant contribution of Z-boson mass has the following expression

$$M_Z^2 \simeq -0.21\mu_0^2 + 7.7m_{1/2}^2 + 1.91m_0^2 + 0.08A_{t_0}^2 + 0.024A_{\kappa_{10_0}}^2 - 0.24m_{1/2}A_{t_0} + 0.12m_{1/2}A_{\kappa_{10_0}} - 0.094A_{t_0}A_{\kappa_{10_0}}. \quad (65)$$

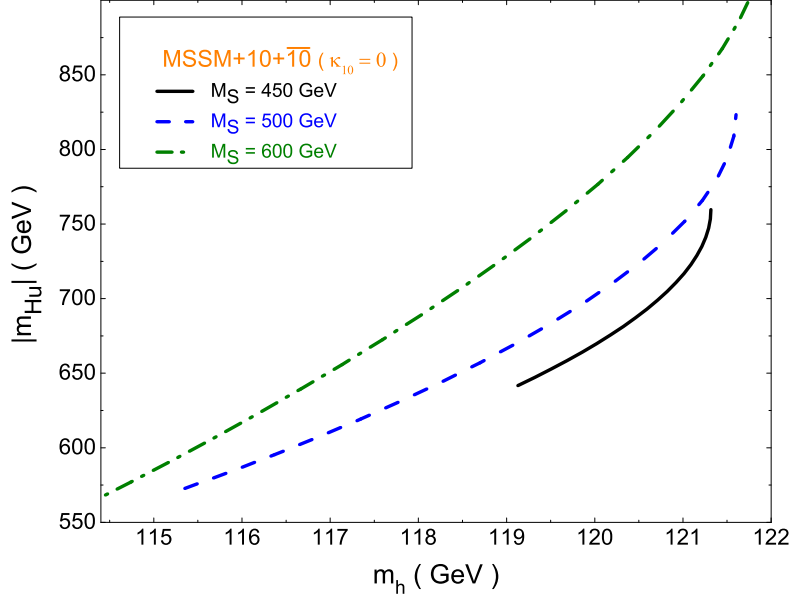


Figure 12:  $|m_{Hu}|$  versus  $m_h$  for  $MSSM+10 + \overline{10}$ , with  $(A_t < 0)$ . Solid, dashed and dashed-dotted curve correspond to  $M_S = 450, 500$  and  $600$  GeV respectively. For all cases  $\kappa_{10} = 0$  at  $M_G$ .

We see that the coefficient of  $m_{1/2}^2$  in this expression has increased as compared to MSSM case (see Eq.(48)), and so we expect fine tuning to be worse in this case. This is related to the value of  $\kappa_{10}(M_G)$ . If we reduce  $\kappa_{10}(M_G)$ ,  $y_t(M_G)$  is reduced (see Figure (10)), and as a result the coefficient of  $m_{1/2}^2$  in the  $M_Z^2$  expression is reduced. We find that for some values of  $\kappa_{10}(M_G)$  the top Yukawa coupling  $y_t(M_G)$  is smaller than  $y_t^{MSSM}(M_G)$ , its value in the MSSM. This enables us to reduce the degree of fine tuning for the case  $MSSM + 10 + \overline{10}$  compared to the MSSM.

However, with smaller values of  $\kappa_{10}(M_G)$ , the value for the Higgs mass  $m_h$  will be lower. Thus, we need to find an optimum value of  $\kappa_{10}(M_G)$ , between 2 and 0, which gives a sufficiently large value for the Higgs mass, but at the same time yields smaller value of  $|m_{Hu}|$ .

*Case II.* We next study the changes brought about by setting  $\kappa_{10}(M_G) = 0$  and by applying the following boundary conditions

$$\{\alpha_G, M_G, y_t(M_G), \kappa_{10}(M_G), A_{\kappa_{10}}(M_G)\} = \{1/8.55, 2.0 \times 10^{16}, 0.163, 0, 0\}. \quad (66)$$

For the MSSM spectrum and related quantities, we find

$$-m_{Hu}^2(M_Z) = 2.59m_{1/2}^2 - 0.15m_0^2 + 0.12A_{t_0}^2 - 0.76m_{1/2}A_{t_0} \quad (67)$$

$$m_{Qt}^2(M_Z) = 2.64m_{1/2}^2 + 0.72m_0^2 - 0.041A_{t_0}^2 + 0.25m_{1/2}A_{t_0} \quad (68)$$

$$m_{Ut}^2(M_Z) = 1.35m_{1/2}^2 + 0.44m_0^2 - 0.082A_{t_0}^2 + 0.51m_{1/2}A_{t_0} \quad (69)$$

$$A_t(M_Z) = -2.14m_{1/2} + 0.44A_{t_0} \quad (70)$$

$$A_{\kappa_{10}}(M_Z) = -3.02m_{1/2} - 0.28A_{t_0} + A_{\kappa_{10_0}} \quad (71)$$

$$m_{Q_{1,2}}^2(M_Z) = 3.63m_{1/2}^2 + m_0^2 \quad (72)$$

$$m_{U_{1,2}}^2(M_Z) = 3.33m_{1/2}^2 + m_0^2 \quad (73)$$

$$\mu^2(M_Z) = 1.9\mu_0^2 \quad (74)$$

$$m_{D_{1,2,3}}^2(M_Z) = m_0^2 + 3.29m_{1/2}^2 \quad (75)$$

$$m_{L_{1,2,3}}^2(M_Z) = m_0^2 + 0.374m_{1/2}^2 \quad (76)$$

$$m_{E_{1,2,3}}^2(M_Z) = m_0^2 + 0.122m_{1/2}^2 \quad (77)$$

$$m_{Q_{10}}^2(M_Z) = 3.63m_{1/2}^2 + m_0^2 \quad (78)$$

$$m_{U_{10}}^2(M_Z) = 3.33m_{1/2}^2 + m_0^2 \quad (79)$$

$$m_{E_{10}}^2(M_Z) = m_0^2 + 0.122m_{1/2}^2. \quad (80)$$

The dominant contribution of  $Z$ -boson mass has the following expression

$$M_Z^2 \simeq -3.78\mu_0^2 + 5.19m_{1/2}^2 - 0.31m_0^2 + 0.25A_{t_0}^2 - 1.52m_{1/2}A_{t_0}. \quad (81)$$

In Figure 11 we plot  $|m_{H_U}|$  versus the lightest CP even Higgs mass  $m_h$ , with  $M_S = 600$  GeV. We compare two cases, when  $\kappa_{10} = 1.1$  and  $\kappa_{10} = 0$  at  $M_G$ . The case  $\kappa = 1.1$  is interesting in the sense that in this case the value of top Yukawa coupling is the same as in the MSSM. We can see in Figure 11 that for a Higgs mass less than 118 GeV, the fine tuning responsible for little hierarchy problem is less severe, while larger than these values the situation becomes worse. We do not display the case  $\kappa_{10} = 2$  for which  $|m_{H_U}|$  exceeds 2 TeV. On the other hand one can see how fine tuning condition for little hierarchy problem is improved when  $\kappa = 0$  at GUT scale.

In Figure 12 we plot  $|m_{H_U}|$  versus the  $m_h$  for MSSM+10 +  $\overline{10}$ , with  $A_t < 0$ . Solid, dashed and dashed-dotted curve correspond to  $M_S = 450, 500$  and  $600$  GeV respectively. For all cases  $\kappa_{10} = 0$  at  $M_G$ . We see that the fine tuning condition is relaxed compared to the results in Figure 9.

### 4.3 MSSM+5 + $\overline{5}$

In this section we consider MSSM+5 +  $\overline{5}$  and perform an analysis similar to what we did for MSSM+10 +  $\overline{10}$ .

*Case I.* Using the boundary conditions

$$\{\alpha_G, M_G, y_t(M_G), \kappa_5(M_G)\} = \{1/19.06, 2.0 \times 10^{16}, 0.57, 2\}, \quad (82)$$

we obtain the following spectrum

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} = \{0.323, 0.645, 2.23\}m_{1/2} \quad (83)$$

and

$$\begin{aligned} M_Z^2 = & -1.26\mu_0^2 + 5.32m_{1/2}^2 + 0.94m_0^2 + 0.18A_{t_0}^2 + 0.04A_{\kappa_{5_0}}^2 \\ & -0.86m_{1/2}A_{t_0} + 0.21m_{1/2}A_{\kappa_{5_0}} - 0.09A_{t_0}A_{\kappa_{5_0}} \end{aligned} \quad (84)$$

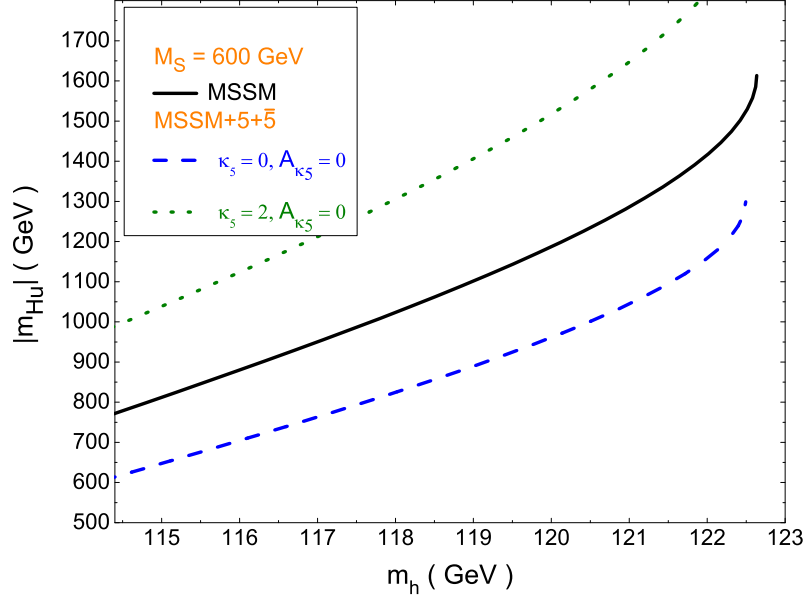


Figure 13:  $|m_{H_u}|$  versus  $m_h$  for MSSM+5 +  $\bar{5}$  ( $A_t < 0$ ).

$$-m_{H_u}^2(M_Z) = 2.66m_{1/2}^2 + 0.47m_0^2 + 0.09A_{t_0}^2 + 0.02A_{\kappa_{50}}^2 - 0.43m_{1/2}A_{t_0} + 0.1m_{1/2}A_{\kappa_{50}} - 0.044A_{t_0}A_{\kappa_{50}} \quad (85)$$

$$m_{Q_t}^2(M_Z) = 4.68m_{1/2}^2 + 0.7m_0^2 - 0.04A_{t_0}^2 + 0.003A_{\kappa_{50}}^2 + 0.16m_{1/2}A_{t_0} - 0.04m_{1/2}A_{\kappa_{50}} + 0.013A_{t_0}A_{\kappa_{50}} \quad (86)$$

$$m_{U_t}^2(M_Z) = 3.24m_{1/2}^2 + 0.39m_0^2 - 0.072A_{t_0}^2 - 0.006A_{\kappa_{50}}^2 + 0.32m_{1/2}A_{t_0} - 0.08m_{1/2}A_{\kappa_{50}} + 0.026A_{t_0}A_{\kappa_{50}} \quad (87)$$

$$A_t(M_Z) = -2.17m_{1/2} + 0.28A_{t_0} - 0.076A_{\kappa_{50}} \quad (88)$$

$$A_{\kappa_5}(M_Z) = 0.36m_{1/2} - 0.21A_{t_0} + 0.16A_{\kappa_{50}} \quad (89)$$

$$m_{Q_{1,2}}^2(M_Z) = 5.73m_{1/2}^2 + m_0^2 \quad (90)$$

$$m_{U_{1,2}}^2(M_Z) = 5.36m_{1/2}^2 + m_0^2 \quad (91)$$

$$\mu^2(M_Z) = 0.63\mu_0^2 \quad (92)$$

$$m_{D_{1,2,3}}^2(M_Z) = m_0^2 + 5.31m_{1/2}^2 \quad (93)$$

$$m_{L_{1,2,3}}^2(M_Z) = m_0^2 + 0.474m_{1/2}^2 \quad (94)$$

$$m_{E_{1,2,3}}^2(M_Z) = m_0^2 + 0.141m_{1/2}^2. \quad (95)$$

For those particles outside the the MSSM, we find

$$m_{L_5}^2(M_Z) = 0.51m_{1/2}^2 + 0.45m_0^2 + 0.02A_{t_0}^2 - 0.03A_{\kappa_{50}}^2 - 0.042m_{1/2}A_{t_0} + 0.015m_{1/2}A_{\kappa_{50}} + 0.005A_{t_0}A_{\kappa_{50}}. \quad (96)$$

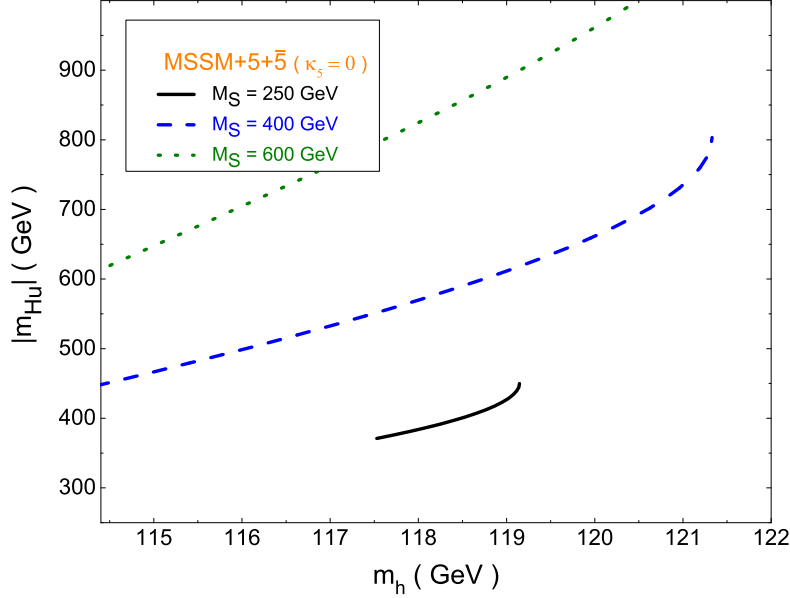


Figure 14:  $|m_{H_u}|$  versus  $m_h$  for MSSM+5 +  $\bar{5}$  ( $A_t < 0$ )

$$m_S^2(M_Z) = 0.073m_{1/2}^2 - 0.11m_0^2 + 0.037A_{t_0}^2 - 0.06A_{\kappa_{5_0}}^2 - 0.088m_{1/2}A_{t_0} + 0.03m_{1/2}A_{\kappa_{5_0}} + 0.01A_{t_0}A_{\kappa_{5_0}} \quad (97)$$

$$m_{D_5}^2(M_Z) = m_0^2 + 5.31m_{1/2}^2. \quad (98)$$

*Case II.* Employing the boundary conditions

$$\{\alpha_G, M_G, y_t(M_G), \kappa_5(M_G)\} = \{1/19.06, 2.0 \times 10^{16}, 0.39, 0\}. \quad (99)$$

we find

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} = \{0.323, 0.645, 2.23\}m_{1/2} \quad (100)$$

and

$$M_Z^2 = -2.41\mu_0^2 + 5.37m_{1/2}^2 + 0.03m_0^2 + 0.22A_{t_0}^2 - 1.1m_{1/2}A_{t_0} \quad (101)$$

$$m_{H_u}^2(M_Z) = 2.68m_{1/2}^2 + 0.014m_0^2 + 0.11A_{t_0}^2 - 0.53m_{1/2}A_{t_0} \quad (102)$$

$$m_{Q_t}^2(M_Z) = 4.68m_{1/2}^2 + 0.66m_0^2 - 0.04A_{t_0}^2 + 0.18m_{1/2}A_{t_0} \quad (103)$$

$$m_{U_t}^2(M_Z) = 3.25m_{1/2}^2 + 0.32m_0^2 - 0.073A_{t_0}^2 + 0.35m_{1/2}A_{t_0} \quad (104)$$

$$A_t(M_Z) = -2.25m_{1/2} + 0.32A_{t_0} \quad (105)$$

$$A_{\kappa_5}(M_Z) = 0.23m_{1/2} - 0.34A_{t_0} + A_{\kappa_{5_0}} \quad (106)$$

$$m_{Q_{1,2}}^2(M_Z) = 5.73m_{1/2}^2 + m_0^2 \quad (107)$$

$$m_{U_{1,2}}^2(M_Z) = 5.36m_{1/2}^2 + m_0^2 \quad (108)$$

$$\mu^2(M_Z) = 1.20\mu_0^2 \quad (109)$$

$$m_{D_{1,2,3}}^2(M_Z) = m_0^2 + 5.31m_{1/2}^2 \quad (110)$$

$$m_{L_{1,2,3}}^2(M_Z) = m_0^2 + 0.474m_{1/2}^2 \quad (111)$$

$$m_{E_{1,2,3}}^2(M_Z) = m_0^2 + 0.141m_{1/2}^2 \quad (112)$$

Similarly,

$$m_{L_5}^2(M_Z) = 0.47m_{1/2}^2 + m_0^2 \quad (113)$$

$$m_S^2(M_Z) = m_0^2 \quad (114)$$

$$m_{D_5}^2(M_Z) = m_0^2 + 5.31m_{1/2}^2. \quad (115)$$

In Figure 13 we plot  $|m_{H_U}|$  versus  $m_h$ , with  $M_S = 600$  GeV. We consider two cases,  $\kappa_5 = 2$  and  $\kappa_5 = 0$  at  $M_G$ . We can see from Figure 13 that the fine tuning condition for the little hierarchy problem improves relative to MSSM when  $\kappa_5 = 0$ , but improvement is not as significant as for MSSM+10 +  $\overline{10}$ . The reason is that the values for  $y_t$  at  $M_G$  with  $\kappa_5 = 0$  is large compared to  $y_t$  for MSSM+10 +  $\overline{10}$  with  $\kappa_{10} = 0$ .

In Figure 14 we plot  $|m_{H_U}|$  versus  $m_h$  for MSSM+5 +  $\overline{5}$ , with  $(A_t < 0)$ . Solid, dashed and dashed-dotted curve correspond to  $M_S = 250, 400$  and  $600$  GeV respectively, with  $\kappa_5 = 0$  at  $M_G$ . We see that the fine tuning condition is relaxed compared to the results in Figure 9.

## 5 Conclusion

We have shown that in an extended MSSM framework with vectorlike supermultiplets, whose masses lie in the TeV range, the mass of the lightest CP even Higgs boson can be as high as 160 GeV. Gauge coupling unification is maintained in this approach with the three MSSM gauge couplings remaining perturbative all the way to the GUT scale  $M_G$ . As far as the little hierarchy problem is concerned, the degree of fine tuning in this extended MSSM is severe for larger values of the Higgs mass. However, things have improved somewhat compared to the MSSM if the Higgs mass is found to be  $\lesssim 125$  GeV.

## 6 Acknowledgements

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## 7 Appendix

### A RGEs for MSSM+10 + $\overline{10}$

$$\begin{aligned}
W &= y_t Q_t \overline{U}_t H_u + \kappa_{10} Q_{10} \overline{U}_{10} H_u \\
\frac{d\overline{\alpha}_i}{dt} &= -b_i \overline{\alpha}_i^2 \\
\frac{dM_i}{dt} &= -b_i \overline{\alpha}_i M_i \\
\text{Here } t &= \log \left( \frac{M_G^2}{Q^2} \right), \overline{\alpha}_i = \frac{\alpha_i}{4\pi} \\
\frac{dm_L^2}{dt} &= \left( 3\overline{\alpha}_2 M_2^2 + \frac{3}{5}\overline{\alpha}_1 M_1^2 \right) \\
\frac{dm_E^2}{dt} &= \left( \frac{12}{5}\overline{\alpha}_1 M_1^2 \right) \\
\frac{dm_{Q_t}^2}{dt} &= \left( \frac{16}{3}\overline{\alpha}_3 M_3^2 + 3\overline{\alpha}_2 M_2^2 + \frac{1}{15}\overline{\alpha}_1 M_1^2 \right) \\
&\quad - Y_t (m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2) \\
\frac{dm_{U_t}^2}{dt} &= \left( \frac{16}{3}\overline{\alpha}_3 M_3^2 + \frac{16}{15}\overline{\alpha}_1 M_1^2 \right) - 2Y_t (m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2) \\
\frac{d\mu^2}{dt} &= 3 \left( \overline{\alpha}_2 + \frac{1}{5}\overline{\alpha}_1 \right) \mu^2 - 3(Y_t + K_{10}) \mu^2 \\
\frac{dm_{H_d}^2}{dt} &= 3 \left( \overline{\alpha}_2 M_2^2 + \frac{1}{5}\overline{\alpha}_1 M_1^2 \right) \\
\frac{dm_{H_u}^2}{dt} &= 3 \left( \overline{\alpha}_2 M_2^2 + \frac{1}{5}\overline{\alpha}_1 M_1^2 \right) - 3Y_t (m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2) \\
&\quad - 3K_{10} (m_{Q_{10}}^2 + m_{U_{10}}^2 + m_{H_u}^2 + A_{\kappa_{10}}^2) \\
\frac{dA_t}{dt} &= - \left( \frac{16}{3}\overline{\alpha}_3 M_3 + 3\overline{\alpha}_2 M_2 + \frac{13}{15}\overline{\alpha}_1 M_1 \right) - 6Y_t A_t - 3K_{10} A_{\kappa_{10}} \\
\frac{dY_t}{dt} &= Y_t \left( \frac{16}{3}\overline{\alpha}_3 + 3\overline{\alpha}_2 + \frac{13}{15}\overline{\alpha}_1 \right) - 6Y_t^2 - 3Y_t K_{10} \\
\frac{dm_{Q_{10}}^2}{dt} &= \left( \frac{16}{3}\overline{\alpha}_3 M_3^2 + 3\overline{\alpha}_2 M_2^2 + \frac{1}{15}\overline{\alpha}_1 M_1^2 \right) \\
&\quad - K_{10} (m_{Q_{10}}^2 + m_{U_{10}}^2 + m_{H_u}^2 + A_{\kappa_{10}}^2) \\
\frac{dm_{U_{10}}^2}{dt} &= \left( \frac{16}{3}\overline{\alpha}_3 M_3^2 + \frac{16}{15}\overline{\alpha}_1 M_1^2 \right) - 2K_{10} (m_{Q_{10}}^2 + m_{U_{10}}^2 + m_{H_u}^2 + A_{\kappa_{10}}^2) \\
\frac{dA_{\kappa_{10}}}{dt} &= - \left( \frac{16}{3}\overline{\alpha}_3 M_3 + 3\overline{\alpha}_2 M_2 + \frac{13}{15}\overline{\alpha}_1 M_1 \right) - 6K_{10} A_{\kappa_{10}} - 3Y_t A_t
\end{aligned}$$



$$\begin{aligned}
\frac{dK_{10}}{dt} &= K_{10} \left( \frac{16}{3}\bar{\alpha}_3 + 3\bar{\alpha}_2 + \frac{13}{15}\bar{\alpha}_1 \right) - 6K_{10}^2 - 3Y_t K_{10} \\
\frac{dm_D^2}{dt} &= \left( \frac{16}{3}\bar{\alpha}_3 M_3^2 + \frac{4}{15}\bar{\alpha}_1 M_1^2 \right). \\
\text{Here } Y_t &= \frac{y_t^2}{(4\pi)^2}, \quad K_{10} = \frac{\kappa_{10}^2}{(4\pi)^2}, \quad b_i = \left\{ \frac{33}{5}, 1, -3 \right\} + \{3, 3, 3\}.
\end{aligned}$$

## B RGEs for MSSM+5 + $\bar{5}$

$$\begin{aligned}
W &= y_t Q_t \bar{U}_t H_u + \kappa_5 L_5 \bar{S} H_u \\
\frac{dm_L^2}{dt} &= 3 \left( \bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2 \right) \\
\frac{dm_E^2}{dt} &= \left( \frac{12}{5}\bar{\alpha}_1 M_1^2 \right) \\
\frac{dm_{Q_t}^2}{dt} &= \left( \frac{16}{3}\bar{\alpha}_3 M_3^2 + 3\bar{\alpha}_2 M_2^2 + \frac{1}{15}\bar{\alpha}_1 M_1^2 \right) \\
&\quad - Y_t (m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2) \\
\frac{dm_{U_t}^2}{dt} &= \left( \frac{16}{3}\bar{\alpha}_3 M_3^2 + \frac{16}{15}\bar{\alpha}_1 M_1^2 \right) - 2Y_t (m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2) \\
\frac{d\mu^2}{dt} &= 3 \left( \bar{\alpha}_2 + \frac{1}{5}\bar{\alpha}_1 \right) \mu^2 - (3Y_t + K_5) \mu^2 \\
\frac{dm_{H_d}^2}{dt} &= 3 \left( \bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2 \right) \\
\frac{dm_{H_u}^2}{dt} &= 3 \left( \bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2 \right) - 3Y_t (m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2) \\
&\quad - K_5 (m_{L_5}^2 + m_S^2 + m_{H_u}^2 + A_{\kappa_5}^2) \\
\frac{dA_t}{dt} &= - \left( \frac{16}{3}\bar{\alpha}_3 M_3 + 3\bar{\alpha}_2 M_2 + \frac{13}{15}\bar{\alpha}_1 M_1 \right) - 6Y_t A_t - K_5 A_{\kappa_5} \\
\frac{dY_t}{dt} &= Y_t \left( \frac{16}{3}\bar{\alpha}_3 + 3\bar{\alpha}_2 + \frac{13}{15}\bar{\alpha}_1 \right) - 6Y_t^2 - Y_t K_5 \\
\frac{dm_{L_5}^2}{dt} &= 3 \left( \bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2 \right) - K_5 (m_{L_5}^2 + m_S^2 + m_{H_u}^2 + A_{\kappa_5}^2) \\
\frac{dm_S^2}{dt} &= -2K_5 (m_{L_5}^2 + m_S^2 + m_{H_u}^2 + A_{\kappa_5}^2) \\
\frac{dA_{\kappa_5}}{dt} &= -3 \left( \bar{\alpha}_2 M_2 + \frac{1}{5}\bar{\alpha}_1 M_1 \right) - 4K_5 A_{\kappa_5} - 3Y_t A_t \\
\frac{dK_5}{dt} &= 3K_5 \left( \bar{\alpha}_2 + \frac{1}{5}\bar{\alpha}_1 \right) - 4K_5^2 - 3Y_t K_5
\end{aligned}$$

$$\frac{dm_D^2}{dt} = \left( \frac{16}{3}\bar{\alpha}_3 M_3^2 + \frac{4}{15}\bar{\alpha}_1 M_1^2 \right).$$

Here  $Y_t = \frac{y_t^2}{(4\pi)^2}$ ,  $K_5 = \frac{\kappa_5^2}{(4\pi)^2}$ ,  $b_i = \left\{ \frac{33}{5}, 1, -3 \right\} + \{1, 1, 1\}$ .

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